

Ferromagnetic insulator-based superconducting junctions as sensitive electron thermometers

F. Giazotto,^{1,*} P. Solinas,² A. Braggio,^{2,3} and F. S. Bergeret^{4,5,†}

¹NEST Istituto Nanoscienze-CNR and Scuola Normale Superiore, I-56127 Pisa, Italy

²SPIN-CNR, Via Dodecaneso 33, 16146 Genova, Italy

³I.N.F.N. Sezione di Genova Via Dodecaneso 33, 16146, Genova, Italy

⁴Centro de Física de Materiales (CFM-MPC), Centro Mixto CSIC-UPV/EHU, Manuel de Lardizabal 4, E-20018 San Sebastián, Spain

⁵Donostia International Physics Center (DIPC), Manuel de Lardizabal 5, E-20018 San Sebastián, Spain

We present an exhaustive theoretical analysis of charge and thermoelectric transport in a normal metal-ferromagnetic insulator-superconductor (NFIS) junction, and explore the possibility of its use as a sensitive thermometer. We investigated the transfer functions and the intrinsic noise performance for different measurement configurations. A common feature of all configurations is that the best temperature noise performance is obtained in the non-linear temperature regime for a structure based on an europium chalcogenide ferromagnetic insulator in contact with a superconducting Al film structure. For an open-circuit configuration, although the maximal intrinsic temperature sensitivity can achieve $10\text{nKHz}^{-1/2}$, a realistic amplifying chain will reduce the sensitivity up to $10\mu\text{KHz}^{-1/2}$. To overcome this limitation we propose a measurement scheme in a closed-circuit configuration based on state-of-art SQUID detection technology in an inductive setup. In such a case we show that temperature noise can be as low as $35\text{nK Hz}^{-1/2}$. We also discuss a temperature-to-frequency converter where the obtained thermo-voltage developed over a Josephson junction operated in the dissipative regime is converted into a high-frequency signal. We predict that the structure can generate frequencies up to $\sim 120\text{GHz}$, and transfer functions up to 200GHz/K at around $\sim 1\text{K}$. If operated as electron thermometer, the device may provide temperature noise lower than $35\text{nK Hz}^{-1/2}$ thereby being potentially attractive for radiation sensing applications.

PACS numbers: 74.50.+r, 85.25.-j, 74.25.F-, 72.15.Jf

I. INTRODUCTION

Recent theories have shown that the spin-splitting induced in a superconductor (S) placed in contact with a ferromagnetic insulator (FI) can be exploited in different kinds of spin caloritronic devices such as heat valves^{1,2} or thermoelectric elements³⁻⁶. They can be used as building blocks in phase-coherent thermoelectric transistors⁷, and for the creation of magnetic fields induced by a temperature gradient in Josephson junctions (JJs) due to the thermophase effect⁸. NFIS junctions have been also proposed for efficient electron cooling⁹ of the normal metal N¹⁰. The possible applications of superconductor-ferromagnetic structures for thermoelectrics has been also highlighted in a recent review article.¹¹

In the present work we theoretically analyze charge and thermoelectric transport in a prototype structure based on the FIS building block, and explore its application as an ultra-sensitive electron thermometer¹²⁻¹⁹ and, eventually as a temperature-to-frequency converter. Our system consists of a normal metal-ferromagnetic insulator-superconductor (NFIS) junction, denoted here as the thermoelectric element (TE), which is connected, via the superconducting wires S_1 , to a generic load element, as shown in Fig. 1. A temperature difference localized between the N and S side of the TE induces a thermoelectric signal⁴. We consider three different configurations of the load resistance $R_L = \infty$ (open circuit), $R_L = 0$ (closed circuit) and finite load $R_L = R_{JJ}$ where we close the system over a generic Josephson element, in the dissipative regime, with shunting resistance R_{JJ} . Depending on the configuration the device will operate in different

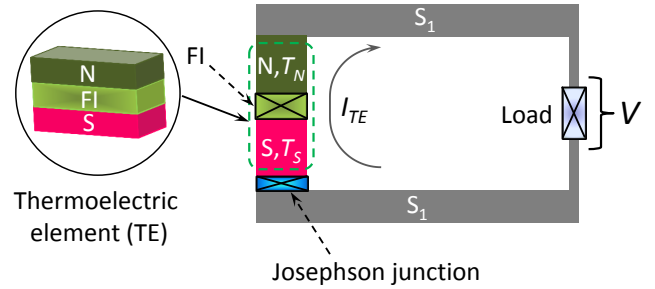


FIG. 1. (Color online) General scheme of the device based on a normal metal-ferromagnetic insulator-superconductor (NFIS) junction. The latter is shown in the blow-up as stacked layers of different materials. T_S and T_N denote the temperature in S and N, respectively, I_{TE} is the thermocurrent circulating in the circuit, and V is the thermovoltage developed across the thermoelectric element (TE). S_1 is a superconductor contacted to both ends of the TE which contains a load with resistance R_L . The latter is intended to be an open-circuit $R_L \rightarrow \infty$, a dissipationless closed-circuit $R_L = 0$, or in the form of a generic Josephson element operated in the dissipative regime in order to convert the thermovoltage V into radiation at the Josephson frequency.

regimes: i) *Seebeck regime*, where a Seebeck thermovoltage (V) is generated across the TE element at open-circuit; ii) *Peltier regime*, where the gradient of temperature generates a circulating thermocurrent that can be probed by an inductive measurement for closed-circuit. Here we explore both regimes that includes an estimate of the intrinsic noise and the best expected temperature sensitivity with state-of-art technology for signal detection. We discuss the advantages and the

drawbacks of the different configurations and show that operated within the non-linear regime, the intrinsic noise of the device is reduced. In particular, our numerical results show that the noise performance is mainly determined by the junction differential resistance R_d which can be drastically reduced beyond the linear-response regime with respect to the temperature. We finally discuss how the generated thermovoltage can induce an ac-Josephson effect with a supercurrent oscillating at a frequency $\nu = |V|/\Phi_0$ ²¹, where $\Phi_0 \simeq 2.067 \times 10^{-15}$ Wb is the flux quantum. The frequency ν can be measured with great accuracy providing accurate and fast information about temperature difference across the TE.

The paper is organized as follows: In Sec. II we briefly present the general formalism and the expressions for the electric current flowing through the NFIS junction and the noise as a function of all the parameters involved in the system. With the help of this expression we analyze in Sec. II A the electric and thermoelectric response of the TE in the non-linear response regime. In particular, we show the impact of the exchange field as well as the role of the barrier polarization on the charge current. In Sec. II B we discuss the different measurement configurations of the device analyzing the effect of the load resistance R_L over the thermo-electrical properties of TE recalling the results for the linear regime in Sec. II C. The evaluation of the intrinsic noise properties of the NFIS junction is done both for the linear and non-linear regime. Assuming a realistic device based on europium sulfide (EuS) as FI and superconducting aluminum (Al), operating at low temperatures, we discuss the open-circuit and closed-circuit configurations respectively in Sec. III and Sec. IV. In those sections we also discuss the temperature noise performance taking into account the most simple measurement scheme with actual state-of-art technologies. Finally, in Sec. V, we discuss the temperature-to-frequency conversion scheme where the thermovoltage developed across the NFIS junction is converted into a high-frequency signal by a Josephson element driven into the dissipative regime. The full temperature-to-frequency conversion capability of the NFIS junction is analyzed, investigating as well the temperature noise performances. We summarize our results in Sec. VI.

II. MODEL

It is instructive to start with the description of the NFIS building block. The interaction between the spin of the conducting electrons in S and the localized magnetic moments in FI lead to an effective exchange interaction in S that decays over the superconducting coherence length ξ_0 ²⁶. We assume that the S layer is thinner than ξ_0 , so that the exchange field (h_{exc}) induced in S by FI is spatially homogenous. In such a case the superconductor density of the states (DoSs) is given by the sum of the densities for spin-up (\uparrow) and spin-down (\downarrow) quasiparticles,

$$N_{\uparrow,\downarrow}(E) = \frac{1}{2} \left| \text{Re} \left[\frac{E + i\Gamma \pm h_{exc}}{\sqrt{(E + i\Gamma \pm h_{exc})^2 - \Delta^2}} \right] \right|. \quad (1)$$

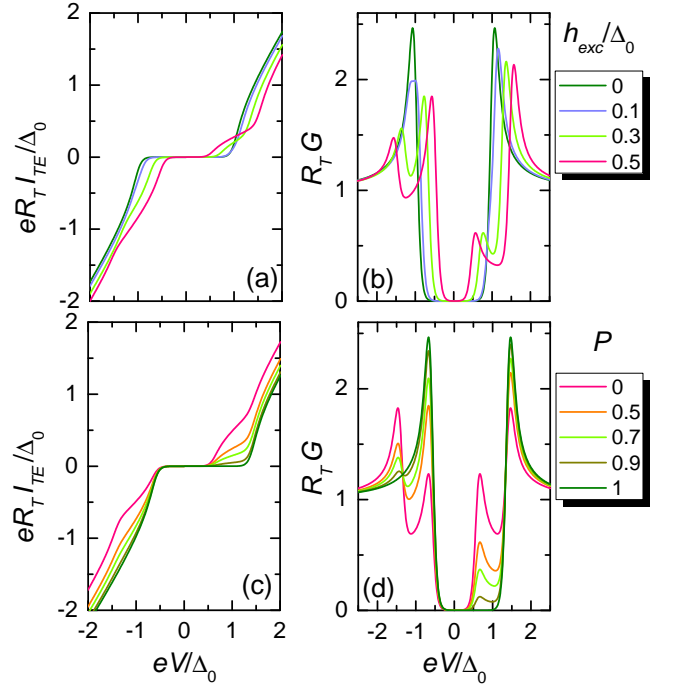


FIG. 2. (Color online) **Characterization of the TE for $T_S = T_N$** (a) Current vs voltage ($I_{TE} - V$) characteristics of the TE element calculated at $T_S = T_N = 0.1T_c$, $P = 0.5$ and for a few values of h_{exc} . (b) Differential conductance vs voltage ($G - V$) characteristics of TE calculated for the same parameters as in panel (a). (c) $I_{TE} - V$ and (d) $G - V$ characteristics of TE calculated at $T_S = T_N = 0.1T_c$, $h_{exc} = 0.4\Delta_0$ and for a few values of P . $\Delta_0 = 1.764k_B T_c$ is the zero-temperature, zero-exchange field superconducting gap, T_c denotes the critical temperature, and R_T is the normal-state resistance of TE.

Here $\Delta(T_S, h_{exc})$ is the pairing potential that depends both on temperature T_S in S and h_{exc} , and it is computed self-consistently in a standard way²² from the gap equation

$$\ln \left(\frac{\Delta_0}{\Delta} \right) = \int_0^{\hbar\omega_D} dE \frac{f_+(E) + f_-(E)}{\sqrt{E^2 + \Delta^2}}, \quad (2)$$

where $f_{\pm}(E) = \left\{ 1 + \exp[\frac{1}{k_B T} (\sqrt{E^2 + \Delta^2} \mp h_{exc})] \right\}^{-1}$, ω_D is the Debye frequency of the superconductor, Δ_0 is the zero-temperature, zero-exchange field superconducting pairing potential, and k_B is the Boltzmann constant. Furthermore, Γ accounts for broadening, and for an ideal superconductor $\Gamma \rightarrow 0$ ²³.

We are interested in the current through the NFIS junction which in the tunneling limit considered here is given by⁴

$$I_{TE} = \frac{1}{eR_T} \int_{-\infty}^{\infty} dE [N_+ + PN_-] [f_N(V, T_N) - f_S(T_S)]. \quad (3)$$

Here R_T is the normal-state resistance of the tunneling junction and $N_{\pm} = (N_{\uparrow} \pm N_{\downarrow})$. Notice that in the tunneling limit the Andreev reflection is negligible small and hence no superconducting proximity effect in N takes place. We assume thermalization on both, the S and the N layer neglecting any deviation of the distribution functions from their equilibrium form²⁰: $f_S(T_S) = [1 + \exp(E/k_B T_S)]^{-1}$ and $f_N(V, T_N) =$

$[1 + \exp[(E + eV)/k_B T_N]]^{-1}$. Here T_N is the temperature in the N layer, and $-e$ is the electron charge. The role of the FI layer is twofold: it acts as a spin filter with polarization²⁴ $P = (G_\uparrow - G_\downarrow)/(G_\uparrow + G_\downarrow)$ and causes the spin-splitting of the DoS in the S layer due to the exchange coupling between the localized magnetic moments of the FI and the conducting electrons of S^{25–27}. These two features have been demonstrated in several experiments^{28,30–34}. Notice that, according to Eq. (3), even in the absence of a voltage bias across the junction a finite current I_{TE} can flow provided $T_N \neq T_S$, as demonstrated in Ref. 4.

A. Electric and thermoelectric response of the TE

Before analyzing the role of a temperature bias across the TE, we determine the current-voltage characteristics (IVCs) and differential conductance $G = dI_{TE}/dV$ of the NFIS junction. We set a low temperature, $T_N = T_S = 0.01T_c$, where $T_c = \Delta_0/(1.764k_B)$ is the critical temperature of the superconductor.

The results obtained from Eq. (3) are summarized in Fig. 2. Panels (a) and (b) show the IVC and G , respectively, for a polarization of the barrier $P = 50\%$ and different values of the spin-splitting exchange field h_{exc} . In panel (a) one clearly sees the deviation of the IVCs from those of a metal-insulator-superconductor (NIS) junction. For finite values of h_{exc} there is a sizeable subgap current [see Fig. 2(a)] as a consequence of the spin-splitting of the DoSs in the S electrode. This splitting manifests itself also in the differential conductance G [see Fig. 2(b)], where the coherent peaks, usually appearing at $V = \pm\Delta/e$, are now split in four peaks appearing at $V = (\pm\Delta \pm h_{exc})/e$. The asymmetry in the height of the coherent peaks stems from the spin polarization P of the FI barrier [see Figs. 2(c,d)] where we set $h_{exc} = 0.4\Delta_0$ and the curves are calculated for different values of P . Therefore from IVCs one can estimate both the polarization of the barrier and the spin-splitting induced in S²⁸.

We now assume a finite temperature difference between the electrodes²⁹, $\delta T = T_S - T_N$, and re-calculate the IVCs from Eq. (3) for $h_{exc} = 0.4\Delta_0$ and $P = 0.9$. The results are shown in Figs. 3(a) and 3(b), where we keep one of the electrodes at temperature $0.01T_c$ and change the other electrode temperature. The curves in Fig. 3 reveal two main properties of the IVC. First, the IVC strongly depends on the amplitude of the temperature difference δT : the larger the temperature difference, the larger is the current flow at low voltages. In the case that the S electrode is heated [see Fig. 3(a)], this trend is limited by the reduced critical temperature $T_c^* < T_c$ of the superconductor originating from the presence of a finite h_{exc} which suppresses the $\Delta(T_S, h_{exc})$ calculated self-consistently. When $T_S \rightarrow T_c^*$, the TE is driven into the normal state with an ohmic characteristic [the red curve in Fig. 3(a)].

Second, there is another interesting feature of the IVCs: they strongly depend on the *sign* of δT . For the same value of $|\delta T|$, the current at $V = 0$ is larger when the N electrode is colder than the S one, i.e., when $\delta T > 0$. In other words, the thermoelectric effect in the TE strongly depends on the

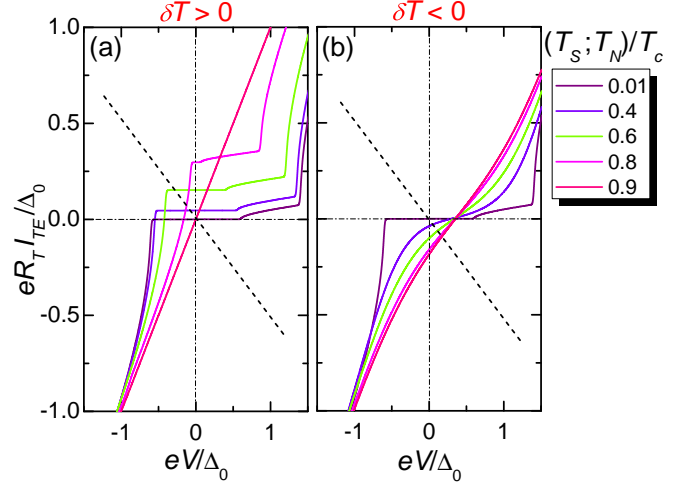


FIG. 3. (Color online) **Characterization of the TE for $T_S \neq T_N$** (a) $I_{TE} - V$ characteristic of TE calculated for several values of T_S (see top legend) at $T_N = 0.01T_c$, $P = 0.9$, and $h_{exc} = 0.4\Delta_0$. (b) The same as in panel (a) calculated for several T_N values at $T_S = 0.01T_c$. Dashed lines in panels (a) and (b) represent the current (I_{JJ}) flowing through the Josephson element when it is operated in the resistive regime, $I_{JJ} = -V/R_{JJ}$.

temperature difference. This feature was not investigated in previous works^{3,4} in which only the linear response regime was discussed.

B. Measurement configurations

To use the TE element as a thermometer we need now to extract from the thermoelectrical signal the temperature gradient present across the TE junction. In order to do so we need to close the TE circuit over a generic load element modeled with a load resistance R_L (see Fig. 1). In such a case the voltage V developed across the TE for a given δT is the solution of the following non-linear integral equation

$$I_{TE}(V, T_S, T_N, h_{exc}, P) + \frac{V}{R_L} = 0, \quad (4)$$

where I_{TE} is defined in Eq. (3). The solution to the above equation is given by the point in which the dashed line (with slope proportional to $1/R_L$) in Figs. 3(a) and 3(b) intersects the IVCs.

For a Seebeck-like measurement one needs to maximize the thermovoltage opening the circuit, i.e., $R_L \rightarrow \infty$ and $I_{TE} = 0$. For a Peltier-like measurement, one needs to maximize the current closing the circuit with a superconducting loop, i.e., $R_L = 0$ and consequently $V = 0$. In the case of temperature-to-frequency conversion that we will discuss later, one needs to include a Josephson element that operates in the dissipative regime with a load resistance $R_L = R_{JJ}$, which is the total shunting resistance of the Josephson element.

Independently of the chosen configuration, we assume to connect the TE to the detector with two superconducting arms

S_1 . In particular, we assume to place a tunnel barrier between S and S_1 to isolate the S element thereby ensuring its description as a thermally homogeneous superconductor with a spin-split DoSs. We neglect here any influence of S_1 arms such that the current through the TE is described by Eq. (3). Superconductors S and S_1 are Josephson coupled through the barrier so that no additional voltage drop will occur. Furthermore, we also assume the NS_1 junction to be a clean metallic contact, thereby contributing negligibly to the total resistance of the system and, for simplicity, we disregard the proximity effect induced into the N layer by the nearby contacted superconductor S_1 ²².

C. Linear response regime

In the linear response regime, i.e., when the voltage V and temperature difference $\delta T \ll T \equiv (T_S + T_N)/2$ across the NFIS junction are small, Eq. (4) reads^{3,4}.

$$I_{TE} \approx I_{TE}^{lin} = \sigma V + P\alpha \frac{\delta T}{T}, \quad (5)$$

where

$$\sigma = \frac{1}{R_T} \int_{-\infty}^{\infty} dE \frac{N_+}{4k_B T \cosh^2\left(\frac{E}{2k_B T}\right)} \quad (6)$$

is the electric conductance, and α is the thermoelectric Seebeck coefficient⁴ defined as

$$\alpha = \frac{1}{eR_T} \int_{-\infty}^{\infty} dE \frac{EN_-}{4k_B T \cosh^2\left(\frac{E}{2k_B T}\right)}. \quad (7)$$

which, in the linear regime, is connected to the Peltier coefficient $\Pi = \alpha/T$ by Onsager symmetry. Substituting Eq. (5) in Eq. (4) and solving respect to the thermovoltage across TE element one finds

$$V^{lin} \simeq -P\alpha \frac{R_L}{R_L \sigma + 1} \frac{\delta T}{T}, \quad (8)$$

that is valid in the linear response regime assuming a generic load resistance R_L . We see immediately that the thermovoltage directly *measures* the temperature gradient in TE. Furthermore, for fixed load resistance, the achievable thermo-voltage V increases with the polarization P . In an open-circuit configuration ($R_L \rightarrow \infty$ and $I_{TE} = 0$) the TE thermo-voltage is maximal being

$$V^{lin} \simeq -\frac{P\alpha}{\sigma} \frac{\delta T}{T}. \quad (9)$$

For the closed-circuit ($R_L = 0$ and $V = 0$) instead the thermocurrent is maximal being

$$I_{TE}^{lin} \approx P\alpha \frac{\delta T}{T}. \quad (10)$$

Obviously we see that in the linear regime the open-circuit thermovoltage V^{lin} is directly related to the closed-circuit thermocurrent, $I^{lin} = \sigma V^{lin}$. In particular, the dependence of the

conversion efficiency on the polarization P and the temperature gradient are the same. This simple picture drastically changes if one goes beyond the linear response regime, i.e., $\delta T \sim T$. We will see below that the non-linear regime is essential in order to optimize the sensitivity for thermometry applications⁹.

D. Intrinsic noise of TE element

We now address the zero frequency noise performance of the NFIS junction. In this case, the main source of noise is the current noise (S_I) generated in the TE that is described by generalizing the expression derived in Ref.³⁷ in the presence of a ferromagnetic tunneling barrier:

$$S_I = \frac{2}{R_T} \int_{-\infty}^{\infty} dE [N_+ + PN_-] \mathcal{M}(E, V, T_N, T_S), \quad (11)$$

where

$$\mathcal{M} = f_N(V, T_N)[1 - f_S(T_S)] + f_S(T_S)[1 - f_N(V, T_N)], \quad (12)$$

and the bias V is given by the solution of Eq. (4). We note that the previous formula describes both thermal, i.e., $eV \ll T_N, T_S$, and shot noise, i.e., $T_N, T_S \ll eV$, and holds in the tunneling regime.

Previous expression simplifies in the linear response regime discussed before where we can neglect any term $\mathcal{O}(\delta T)$ in Eq. (11) finding the thermal noise

$$S_I^{lin} = \frac{4}{R_T} \int_{-\infty}^{\infty} dE [N_+ + PN_-] f_N(T)[1 - f_S(T)], \quad (13)$$

which may be expressed as

$$S_I^{lin} = 4k_B T \sigma, \quad (14)$$

where σ is the TE electric conductance of Eq. (6). In the open-circuit configuration, it is more convenient to write the voltage noise spectral density

$$S_V^{lin} = 4k_B T / \sigma. \quad (15)$$

Below we show that in the non-linear regime one can approximate Eqs. (14)-(15) by substituting σ by $1/R_d$, where R_d is the TE differential resistance.

III. TEMPERATURE-TO-VOLTAGE CONVERSION

When TE is in an *open-circuit* configuration ($R_L \rightarrow \infty$), one can realize a *temperature-to-voltage* conversion scheme. In such case no charge current flows through the TE,

$$I_{TE}(V_0, T_S, T_N, h_{exc}, P) = 0, \quad (16)$$

then a voltage V_0 develops across the TE for $\delta T \neq 0$. The value of V_0 can be obtained from the solution of Eq. (16). The results are shown in the two upper panels of Fig. 4. Specifically, panel 4(a) shows the dependence of V_0 on T_S for different values of h_{exc} , $P = 0.9$ and $T_N = 0.01T_c$. The increase

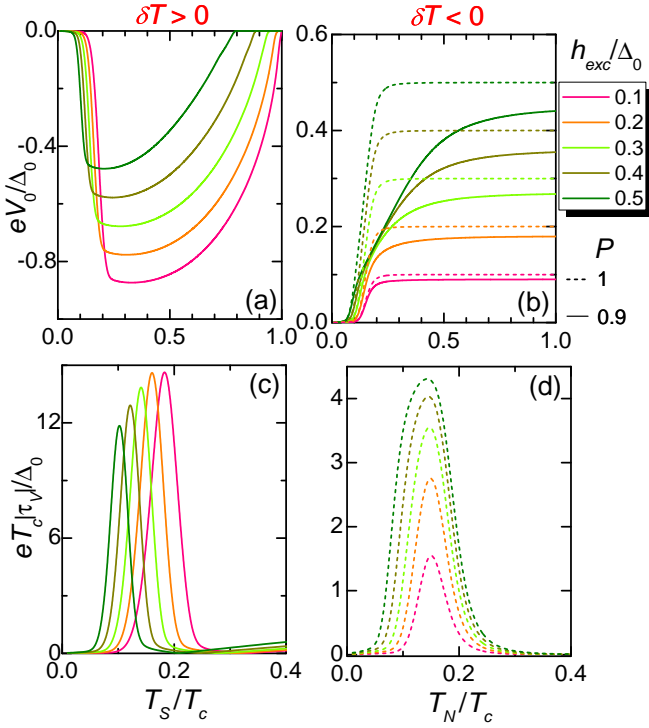


FIG. 4. (Color online) **Open-circuit configuration.** (a) Thermo-voltage V_0 vs T_S calculated for a few values of h_{exc} (see legend) at $T_N = 0.01T_c$ and $P = 0.9$. (b) V_0 vs T_N calculated for a few values of h_{exc} at $T_S = 0.01T_c$ and $P = 0.9$ (full lines), or $P = 1$ (dashed lines). (c) and (d) show the absolute value of the corresponding temperature-to-voltage transfer function $|\tau_V|$ calculated for the same values of h_{exc} .

of T_S , from the minimal temperature T_N , leads first to an enhancement of $|V_0|$. A further increase of T_S leads to the suppression of the superconducting energy gap and a corresponding suppression of V_0 . The voltage V_0 vanishes when superconductivity is fully suppressed for $T_S \rightarrow T_c^*$. We note that V_0 reaches zero continuously owing to the fact that we have chosen values of h_{exc} for which the superconducting-normal state transition is of the second order³⁸

A different temperature behavior of V_0 is obtained when S is kept at $T_S = 0.01T_c$ and T_N is varied, as shown in Fig. 4(b). In particular, besides the obvious change of sign, V_0 grows monotonically by increasing T_N until it reaches an asymptotic value. It is important to stress that the curves $V_0(T_N)$ depends strongly on the polarization P of the barrier [see Fig. 4(b)]. In particular, the larger P the larger is the thermovoltage $V_0(T_N)$ developed across the TE. By contrast, the $V_0(T_S)$ amplitude turns out to be almost unaffected by the value of P .

The different behaviors as a function of δT allow one to reconstruct both the amplitude and direction of the thermal gradient in the TE element. This further information could be eventually exploited to reconstruct the spatial position of a heating event, thereby opening interesting possibilities to build detector-like devices.

An useful figure of merit to estimate the performance of the TE is the temperature-to-voltage transfer function, $\tau_V =$

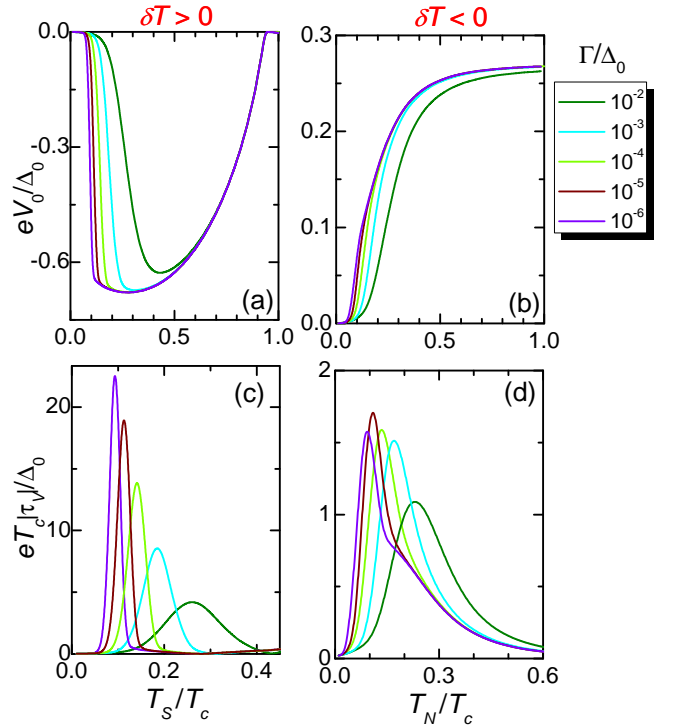


FIG. 5. (Color online) **Open-circuit configuration.** (a) Thermo-voltage V_0 vs T_S calculated for a few values of Γ (see legend) at $T_N = 0.01T_c$, $h_{exc} = 0.3\Delta_0$, and $P = 0.9$. (b) V_0 vs T_N calculated for a few values of Γ at $T_S = 0.01T_c$, $h_{exc} = 0.3\Delta_0$, and $P = 0.9$ (full lines). (c) and (d) show the absolute value of the corresponding temperature-to-voltage transfer function $|\tau_V|$ calculated for the same values of Γ .

$\partial V / \partial T$. The absolute value of this quantity is shown in Fig. 4(c) for $\delta T > 0$ and in Fig. 4(d) for $\delta T < 0$. We have normalized it to the natural unit Δ_0 / eT_c . In Fig. 4(d) we show the case of barrier polarization selectivity $P = 1$ which corresponds to the case with maximal possible transfer function at given h_{exc} .

In order to show the impact of the broadening parameter we display in Fig. 5 the same quantities as in Fig. 4 but calculated for fixed $h_{exc} = 0.3\Delta_0$, $P = 0.9$ and for different values of Γ ranging from $10^{-6}\Delta_0$ up to $10^{-2}\Delta_0$ ^{39–41}. The overall qualitative behavior and the order of magnitude of the effect is the same for all these values. From a quantitative point of view, the temperature-to-voltage conversion turns out to be less effective the larger the value of Γ . Throughout the paper we assume $\Gamma = 10^{-4}\Delta_0$ which is the typical value for conventional Al-based superconducting junctions^{9,39}.

A. Noise performance analysis for the open-circuit configuration

We now focus our analysis on the noise performance of the temperature-to-voltage conversion with the NFIS junction. We need to convert it in a voltage noise assuming that load resistance $R_L \rightarrow \infty$. This means that the voltage noise spectral

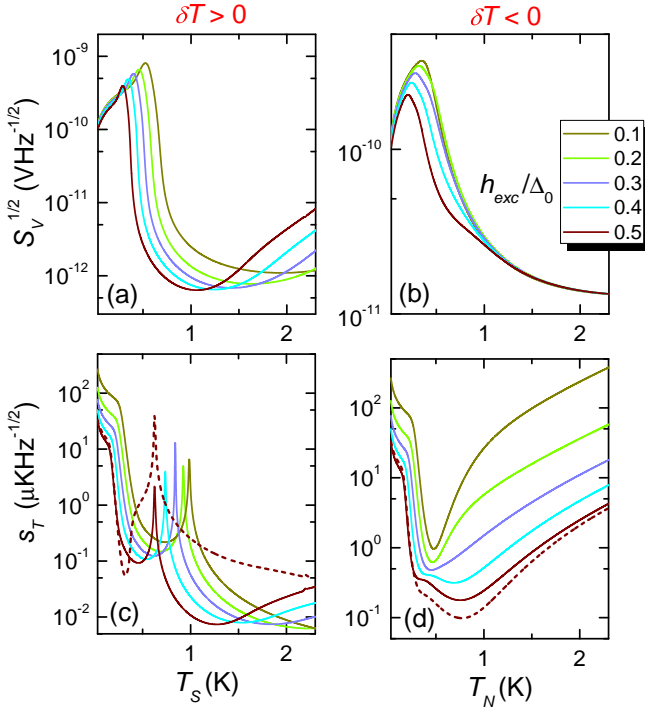


FIG. 6. (Color online) **Open-circuit configuration.** (a) Square root of the voltage noise spectral density $S_V^{1/2}$ vs T_S calculated at $T_N = 0.01T_c$ for a few values of h_{exc} (see legend). (b) $S_V^{1/2}$ vs T_N calculated at $T_S = 0.01T_c$ for the same h_{exc} values. (c) Temperature sensitivity s_T vs T_S calculated for the case of panel (a). (d) s_T vs T_N calculated for the case of panel (b). Dashed lines show s_T as calculated using the approximate formula Eq. 19 for $h_{exc}/\Delta_0 = 0.5$. In all these calculations we set $P = 0.98$, and $T_c = 3K$.

density (S_V) generated across the TE, is³⁶

$$S_V(V_0, T_S, T_N, h_{exc}, P) = S_I R_d^2, \quad (17)$$

where the $R_d = \partial V_0 / \partial I_{TE}$ is the differential resistance of the TE, and the bias V_0 is given by the solution of Eq. (16).

In Fig. 6(a)-(b) the square root of noise spectral density ($\sqrt{S_V}$) is displayed for a TE element with a barrier characterized by a realistic value of polarization $P = 0.98$. This spin-filter efficiency is representative for EuO or EuS FI barriers,³⁵ and we assume a superconductor with $T_c = 3K$ which would be implementable with ultra-thin Al films³⁰⁻³³.

We find that for $\delta T > 0$ the minimal noise value is obtained in the non-linear regime $T_N \ll T_S \lesssim T_c$ where the voltage noise can be as low as $\sim 600 \text{ fV Hz}^{-1/2}$, and is two orders of magnitude lower than the equilibrium case $\delta T/T \ll 1$, where $T = (T_S + T_N)/2$ is the average temperature. For $\delta T < 0$, the noise performance is worse being at best a few tens of $\text{pV Hz}^{-1/2}$ for the non-linear regime $T_S \ll T_N \sim T_c$.

The intrinsic temperature noise (temperature sensitivity) per unit bandwidth of the thermometer (s_T) is related to the voltage noise spectral density as

$$s_T = \frac{\sqrt{S_V}}{|\tau_V|}. \quad (18)$$

In Fig. 6(c)-(d) we show the temperature noise s_T for the open-circuit configuration for the two cases of Fig. 6(a)-(b). The differences between the voltage spectral density is entirely given by the transfer function which is highly non-linear as a function of δT . We notice that in the linear regime $|\delta T|/T \ll 1$ the temperature noise, given by Eq. (15), is a few tens of $\mu\text{K Hz}^{-1/2}$. The maximum temperature sensitivity is obtained in the nonlinear regime, i.e., $|\delta T|/T \gg 1$ where the temperature noise can be as low as $8 \text{ nK Hz}^{-1/2}$ coinciding with the minimal voltage noise. By contrast, for the case $\delta T < 0$, the best noise performance is around $180 \text{ nK Hz}^{-1/2}$.

It is interesting to observe the scaling behaviour of noise power as function of the junction normal-state resistance R_T . Indeed as $S_I \propto 1/R_T$ [see Eq. (11)], from Eq. (17) one can conclude that $S_V \propto R_T$ since $R_d \propto R_T$. At the same time there is no scaling behaviour of the transfer function, $\tau_V = \partial V_0 / \partial T$. This may be easily inferred, for instance, from the expression of the thermovoltage V_0 in the linear regime, Eq. (9), since $V^{lin} \propto \alpha/\sigma$ which is the ratio of two quantities with the same scaling $1/R_T$. Alternatively one can deduce it from the relation between open-circuit voltage and the temperature difference, δT Eq. (16), where R_T enters only as an overall prefactor. We conclude that $s_T \propto \sqrt{R_T}$ which shows immediately that the reduction of TE resistance would be beneficial for increasing the sensitivity in temperature measurement.

These considerations suggest that in the non-linear regime the differential resistance R_d takes the role of R_T . Indeed one can guess a way to generalize Eq. (15) to the non-linear regime by replacing the linear conductance σ by the differential conductance $1/R_d$ such that

$$S_V \approx 4k_B T R_d \quad (19)$$

where the temperature is taken as the average $T = (T_S + T_N)/2$. The previous expression should converge to the linear result when $|\delta T|/T \ll 1$. The full numerical results of Fig. 6(c)-(d) demonstrate the accuracy of Eq. (19) shown as dashed lines. This shows that the noise performance is essentially characterized by the dependence of differential resistance R_d . Therefore this approximation is extremely useful to estimate the noise performance with the knowledge of the differential resistance R_d only.

It is important to emphasize that in a realistic measurement scheme the temperature sensitivity of the device is limited by the amplifying chain. Indeed, in general, the voltage signal must be amplified with a low-noise preamplifier which is characterized by its intrinsic voltage noise. The preamplifier noise may degrade the total noise performance. In particular, assuming for the pre-amp a square root spectral density of $\sim 1 \text{ nV Hz}^{-1/2}$, it is clear that it will dominate over the intrinsic voltage noise of the signal which can be smaller by a few orders of magnitude [see Fig. 6(a)-(b)]. The preamplifier is therefore the main bottleneck to the temperature detection in this configuration scheme although it has the advantage of suppressing the noise non-linearities over the considered temperature window. The realistic performance of this measurement scheme will be roughly $\sim 10 \mu\text{K Hz}^{-1/2}$. This limitation can be overcome by exploiting a close-circuit configuration, as discussed in the next section.

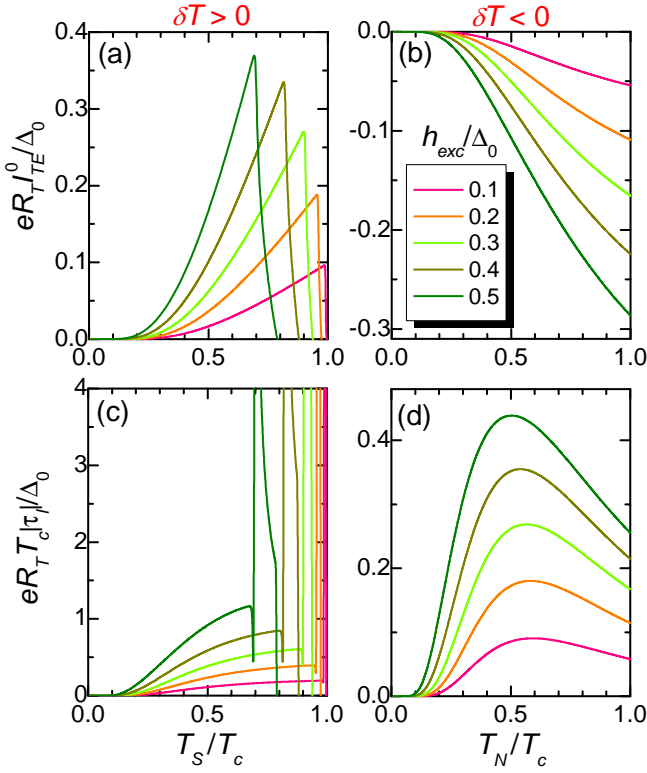


FIG. 7. (Color online) **Closed-circuit configuration** (a) Thermocurrent I_{TE}^0 vs T_S calculated for a few values of h_{exc} at $T_N = 0.01T_c$. (b) I_{TE}^0 vs T_N calculated for the same values of h_{exc} . (c) and (d) show the absolute value of the temperature-to-current transfer function $|\tau_I|$ vs temperature for the panel (a) and (b), respectively, calculated for the same values of h_{exc} . In all these calculations we set $P = 0.98$.

IV. TEMPERATURE-TO-CURRENT CONVERSION

Hereafter we analyse the performance of a closed-circuit configuration which correspond to *temperature-to-current* conversion. In this setup the TE current $I_{TE}(V = 0) = I_{TE}^0$ is given by Eq. (3) which depends only on T_S and T_N .

In Fig. 7(a) we show how the current depends on T_S for different values of h_{exc} keeping fixed the barrier polarization $P = 0.98$ and $T_N = 0.01T_c$. The general behaviour has “shark-fin” shape which increases in amplitude with the h_{exc} . After reaching a maximum at T_S^* the I_{TE}^0 decreases with T_S until the critical temperature T_c^* is reached and the superconductivity is completely suppressed.

If we fix $T_S = 0.01T_c$, by changing T_N we get for $\delta T < 0$ an obvious opposite sign for the thermocurrent I_{TE}^0 , and an absolute value of the thermocurrent $|I_{TE}^0|$ which monotonously increases by enhancing $|\delta T|$. It finally saturates to the maximal value

$$I_{TE,max}^0 = \frac{1}{2eR_T} \int_{-\infty}^{\infty} dE [N_+ + PN_-] \text{sgn}(E), \quad (20)$$

which is easily obtained from the general expression of the TE current, Eq. (3), by taking the limit $T_S \rightarrow 0$ and $T_N \rightarrow \infty$ with $V = 0$. From this, we can conclude that an arbitrary

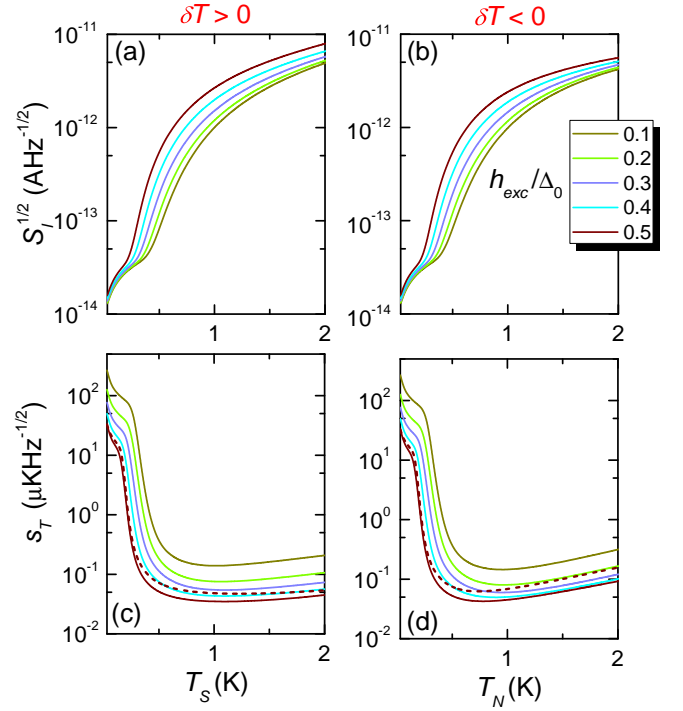


FIG. 8. (Color online) **Closed-circuit configuration** (a) Square root of the current noise spectral density $S_I^{1/2}$ vs T_S for $T_N = 0.01T_c$ and for different values of h_{exc} (see legend). (b) $S_I^{1/2}$ vs T_N for $T_S = 0.01T_c$ for the same values of h_{exc} . (c) s_T vs T_S for the case of panel (a). (d) s_T vs T_N for the case of panel (b). Dashed lines shows s_T as calculated using the approximate formula Eq. (22) for $h_{exc}/\Delta_0 = 0.5$. In all these calculations we set $P = 0.98$ and $T_c = 3K$.

enhancement of $|\delta T|$ is not of particular benefit to increase the current signal.

In Fig. 7(c)-(d) we show the absolute value of the temperature-to-current transfer function $\tau_I = \partial I_{TE}^0 / \partial T$ respectively for the case (a) and (b) of the same figure. For $\delta T > 0$, the transfer function has two different behaviors depending if T_S is smaller or larger than T_S^* . On the other hand one sees that, independently of the sign of δT , the transfer function is maximized in the non-linear regime $|\delta T| \sim T$. This is an important issue in order to increase the temperature sensitivity.

A. Noise performance analysis for the closed-circuit configuration

In Fig. 8(a)-(b) the current noise S_I of the closed-circuit configuration is shown as obtained from Eq. (11) with $V = 0$. The current noise as a function of δT is minimized in the linear regime obtaining $\sim 15 \text{fHz}^{-1/2}$, and grows by increasing $|\delta T|$. The noise behaviour of the closed-circuit configuration is less affected by the sign of δT in comparison to the open-circuit one (see Sec. III A). The current noise increases with h_{exc} since also the average current I_{TE}^0 increases in such a case [see Fig. 7(a) and (b)].

The intrinsic temperature noise per unit bandwidth of the thermometer s_T in this configuration is given by

$$s_T = \frac{\sqrt{S_I}}{|\tau_I|}, \quad (21)$$

where $|\tau_I|$ is the temperature-to-current transfer function discussed before. The same scaling behaviour shown before for $s_T \propto \sqrt{R_T}$ still holds in this configuration since now $S_I \propto 1/R_T$ but $\tau_I \propto 1/R_T$. Consequently also for this case the minimization of R_T would be, in general, beneficial for improving noise performance. As in the previous section one can try to generalise this argument for the non-linear regime by replacing R_T with R_d . Since R_d is largely reduced in comparison to the linear regime value $1/\sigma$, one can expect an increase of the noise in the non-linear regime. At a first glance, this does not look plausible since the current noise is in general higher [see Fig. 8(a)-(b)]. However, our guess seems to be correct, as shown in Fig. 8(c) and (d), where the temperature noise is minimized for large values of δT .

The lowest intrinsic noise $\sim 35 \text{ nK Hz}^{-1/2}$ is obtained in the non-linear regime for $\delta T \approx 1 \text{ K}$, when $P = 0.98$ and $T_c = 3 \text{ K}$ are chosen. The main difference with the open-circuit configuration (see Fig. 6) is that in the present situation the noise depends weakly on the sign of δT in a wide temperature region. Moreover, in contrast to the open-circuit configuration, the noise shows a rather smooth behavior.

As pointed out above, the behaviour of the current noise in the non-linear regime can be approximated by the expression

$$S_I \approx \frac{4k_B T}{R_d}, \quad (22)$$

where the linear conductance σ of Eq. (14) is replaced by the inverse differential resistance, $1/R_d$. In Fig. 8(c) and (d) we show (dashed lines) this approximation for the case where we expect the largest nonlinearities, i.e., for $h_{exc}/\Delta_0 = 0.5$. We can thus conclude that this simple formula gives a fairly accurate description of S_I in the non-linear regime.

In terms of overall temperature noise the closed-circuit configuration has two advantages: First, the smooth behaviour of temperature noise makes it more attractive than the open-circuit configuration. Second, while the ideal noise is better for the open-circuit configuration (see Figs. 6 and 8), one needs to evaluate the total noise of the measurement which includes the addition of the preamplifier noise. The latter, as discussed in the previous section, strongly degrades the resulting noise figure. The close-circuit configuration offers a way to overcome this limitation, as discussed in the following.

Specifically, we propose to measure the current signal by coupling the closed circuit via a mutual inductance M to a SQUID. The latter measures the flux generated by the current circulating in the thermoelectric circuit. The total temperature sensitivity, which includes now the SQUID noise, can be written as

$$s_T^{\text{tot}} = \frac{\sqrt{S_\phi^{\text{TOT}}}}{|\tau_\phi|} = \frac{\sqrt{S_I + (S_\phi^{\text{SQUID}}/M^2)}}{|\tau_I|}, \quad (23)$$

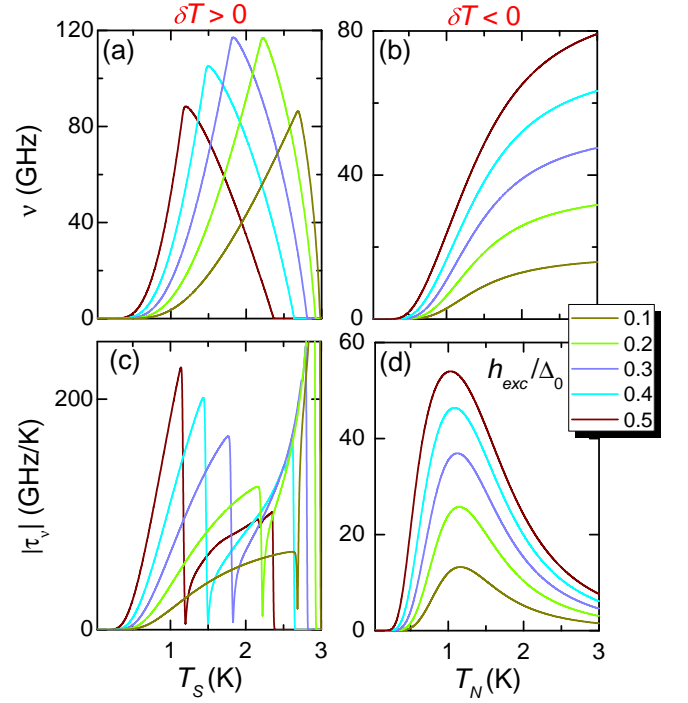


FIG. 9. (Color online) **Characterization of the temperature-to-frequency converter** (a) Frequency ν vs T_S calculated for a few values of h_{exc} at $T_N = 0.01T_c$. (b) ν vs T_N at $T_S = 0.01T_c$ calculated for the same h_{exc} values as in panel (a). (c) and (d) show the absolute value transfer function $|\tau_I|$ of panel (a) and (b), respectively, calculated for the same values of h_{exc} . In all these calculations we set $P = 0.98$, $R_T/R_{JJ} = 0.2$, and $T_c = 3 \text{ K}$.

where the temperature-to-flux transfer function is $\tau_\phi = M\tau_I$, and the TE flux spectral density $S_\phi^{\text{TE}} = M^2 S_I$ is added to the SQUID noise (S_ϕ^{SQUID}) to give the total flux noise, $S_\phi^{\text{TOT}} = S_\phi^{\text{TE}} + S_\phi^{\text{SQUID}}$. The square root of the flux noise for high-quality commercial SQUID can be as low as $\sqrt{S_\phi^{\text{SQUID}}} \sim 10^{-7} \Phi_0 \text{ Hz}^{-1/2}$, which is then converted into an effective circulating current noise in the thermoelectric circuit of $\sim 20 \text{ fA Hz}^{-1/2}$ by dividing it with a typical value for the mutual inductance $M = 10^{-8} \text{ H}$. By looking at Fig. 8(a) and (b) we immediately see that the intrinsic TE current noise will, in general, dominate over the SQUID noise almost everywhere in the non-linear regime where we can achieve the best sensitivity. Therefore, the temperature noise of this measurement scheme is only limited by the intrinsic TE noise mechanisms, and can be as low as $\sim 35 \text{ nK Hz}^{-1/2}$ for a moderate temperature non-linearity (see Fig. 8).

V. TEMPERATURE-TO-FREQUENCY CONVERSION

We now focus on the *temperature-to-frequency* conversion process. This conversion is achieved with the device sketched in Fig. 1 where the thermovoltage generated across the TE is applied to a generic Josephson element. The latter is set to op-

erate in the dissipative regime when $I_{JJ} = V/R_{JJ}$, where R_{JJ} is the total shunting resistance of the Josephson element. In this case, there is a time-oscillating current through the Josephson element with a frequency equal to the Josephson frequency, $\nu = |V|/\Phi_0$. As discussed above, the value of V depends on the temperature difference δT across the TE, and therefore the frequency emitted by the Josephson junction is a measure of δT .

In order to quantify the temperature-to-frequency conversion effect, one has to determine the voltage V developed for any given δT imposed across the TE which satisfies Eq. (4) with a finite load resistance $R_L = R_{JJ}$. This configuration is intermediate between the open-circuit and the closed-circuit setup discussed above. In the following, we consider the case where $R_T/R_{JJ} = 0.2$ in order to produce a detectable frequency signal between 10 GHz and fractions of THz.

The frequency-to-temperature performance of this configuration is shown in Fig. 9. We again used the spin-filter efficiency $P = 0.98$ and the critical temperature $T_c = 3\text{K}$ adopted in the previous sections. Panels 9(a) and 9(b) show the frequency generated by the Josephson element, for positive and negative δT , respectively. In the linear response regime, the TE thermovoltage depends only on $|\delta T|$ as can be seen from Fig. 9. The information about the sign of δT is eventually recovered only for the non-linear regime.

If T_N is kept at $0.01T_c$ the maximum frequency is achieved around $h_{exc} \approx 0.2\Delta_0$ for $T_S \approx 0.75T_c$, and obtains values as large as $\sim 120\text{GHz}$. If T_S is kept at low temperature, ν increases monotonically by increasing both T_N and/or h_{exc} and obtains a maximum of $\sim 80\text{GHz}$.

In the present setup an important figure of merit of the structure is represented by the temperature-to-frequency transfer function, $\tau_v = \partial\nu/\partial T$, plotted in absolute value in Figs. 9(c) and 9(d). In particular, $|\tau_v|$ exceeding 200GHz/K around $T_S \sim 1\text{K}$ can be achieved for $h_{exc} = 0.5\Delta_0$ by heating S, while $|\tau_v|$ up to $\sim 55\text{GHz/K}$ can be achieved with the same values by heating N.

A. Noise performance

In the temperature-to-frequency conversion process, the noise is determined by the bias fluctuations generated from the current noise via the load resistance seen by the TE, i.e., the parallel between the Josephson element total resistance R_{JJ} and the TE resistance R_d : $\mathcal{R} = R_d R_{JJ}/(R_d + R_{JJ})$. Note that the differential resistance $R_d = \partial V_0/\partial I_{TE}$ is calculated from the solutions of Eq. (4) where $R_L = R_{JJ}$. The important quantity is represented by the frequency noise spectral density (S_v) which can be expressed as

$$S_v = \frac{S_I \mathcal{R}^2}{\Phi_0^2}. \quad (24)$$

Finally, the intrinsic temperature noise per unit bandwidth of the thermometer (s_T) is related to the frequency noise spectral

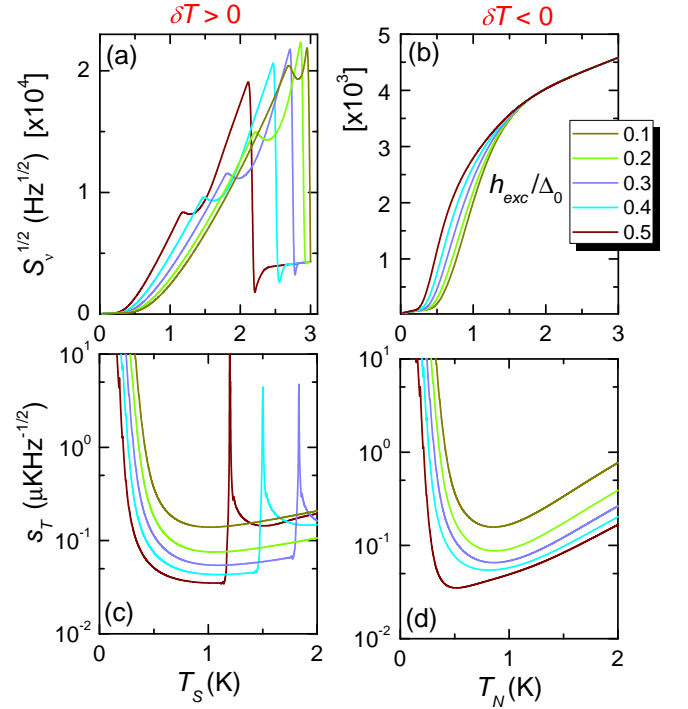


FIG. 10. (Color online) **Noise performance of the temperature-to-frequency converter** (a) Square root of the frequency noise spectral density $S_v^{1/2}$ vs T_S calculated at $T_N = 0.01T_c$ for a few values of h_{exc} . (b) $S_v^{1/2}$ vs T_N calculated at $T_S = 0.01T_c$ for the same h_{exc} values. (c) Temperature sensitivity s_T vs T_S calculated at $T_N = 0.01T_c$ for a few values of h_{exc} . (d) s_T vs T_N calculated at $T_S = 0.01T_c$ for the same h_{exc} values. In all these calculations we set $P = 0.98$, $T_c = 3\text{K}$, $R_T/R_{JJ} = 0.2$, and $R_T = 1\Omega$.

density as

$$s_T = \frac{\sqrt{S_v}}{|\tau_v|}. \quad (25)$$

Figure 10(a) and (b) show the calculated square root of the frequency noise spectral density S_v for positive and negative δT , respectively, calculated for the same parameters as in Fig. 9, and for $R_T = 1\Omega$. In particular, for positive δT , the noise spectrum $S_v^{1/2}$ shows a non-monotonic behavior with a maximum at intermediate temperatures, and suppression at higher δT . By contrast, for $\delta T < 0$ the noise spectrum grows monotonically with $|\delta T|$, and it is less influenced by h_{exc} .

The behavior of s_T is displayed in Fig. 10(c) and (d). At small $|\delta T|$, in the linear regime, the noise sensitivity is given by several tens of $\mu\text{K Hz}^{-1/2}$. By increasing $|\delta T|$ the growth of $S_v^{1/2}$ [Figs. 10(a) and (b)] is advantageously compensated by the enhancement of τ_v [see Fig. 9(c) and (d)]. The best noise performance is obtained when $|\tau_v|$ is quite near its maximum. The values of $s_T \sim 35\text{nK Hz}^{-1/2}$ is obtained around 1K for $h_{exc} = 0.5\Delta_0$. After the minimum of s_T , for $\delta T > 0$, we see a peak due to the divergence of $|\tau_v|^{-1}$, i.e., the vanishing of the transfer function, as shown in Fig. 9(c). Differently, for $\delta T < 0$ one observes a smooth increase of s_T

determined by the progressive reduction of the transfer function [see Fig. 9(d)] which is consequence of the saturation of the frequency when $T_N \rightarrow \infty$. We conclude by noticing that, also for this case, the best noise performance is obtained in the non-linear regime for $|\delta T| \lesssim 1\text{K}$ where s_T is almost independent of the sign of δT . This is essentially due again to the fact that R_d is strongly reduced in the non-linear regime.

In this configuration the power of the generated frequency signal might be somewhat low. Anyway, one can deploy the standard techniques in order to increase the emission power by connecting in parallel arrays of JJs.^{42,43} The only limiting factor in that case would be the power that the TE element could sustain and transfer to the JJs. A rough estimate shows that when $R_T/R_{JJ} = 0.2$ and $R_T = 1\Omega$ the TE could produce a power of the order of $\sim 100\text{pW} \dots 10\text{nW}$ which would be high enough to make the 10 – 100GHz signal generated by the Josephson junction detectable.

VI. SUMMARY

In summary, we have theoretically investigated a thermoelectric structure based on a normal metal-ferromagnetic insulator-superconductor (NFIS) junction. We fully characterize the thermoelectrical properties of the TE both in the linear and non-linear regimes. We assumed different measurement configuration as determined by the load resistance value. In particular, we showed that by exploiting realistic materials such as EuS or EuO (providing polarization P up to $\sim 98\%$) in combination with superconducting Al thin films the device is able to provide remarkable temperature noise performances. We find that in the open circuit configuration, where the temperature signal is returned via the *Seebeck thermo-voltage*, the lowest achievable intrinsic noise of $\sim 10\text{nKHz}^{-1/2}$ is limited by the amplifying chain. On the other side, we found that in the closed-circuit configuration, where the temperature information is encoded in the *Peltier thermo-current*, one can detect the signal via a low-noise flux measurement of an inductively-coupled SQUID. In such case the temperature noise performance is mainly determined by intrinsic noise mechanisms, with the best value of $\sim 35\text{nKHz}^{-1/2}$ achievable with state-of-art SQUID technology. Interestingly, we identified in the differential resistance R_d of the TE one of the main factor that determines the intrinsic

noise performance of the system. This explain why the best noise performances are obtained in the non-linear temperature regime since for that regime R_d is strongly suppressed. This is a non-trivial consequence of the strong non-linearities peculiar of the NIFS junction.

We finally discuss a *temperature-to-frequency* converter where the obtained thermovoltage is converted through a dissipative Josephson junction into a high frequency signal in the frequency window spanning from a few GHz up to $\sim 10^{11}\text{Hz}$. In particular, we have shown that the device allows for the generation of Josephson radiation at a frequency that depends on both the *amplitude* and *sign* of the temperature difference across the NFIS junction therefore opening the route for high-frequency detection associated to high temperature sensitivity. Frequencies up to $\sim 120\text{GHz}$ and large transfer functions (i.e., up to 200GHz/K) around $\sim 1 - 2\text{K}$ can be obtained in a structure implementable with the above mentioned prototype FIs. In this configuration the device is capable to provide intrinsic temperature noise down to $\sim 35\text{nKHz}^{-1/2}$ around 1K for a sufficiently large h_{exc} . The proposed superconducting hybrid structure has the potential for the realization of effective on-demand on-chip temperature-to-frequency converters as well as ultrasensitive electron thermometers or radiation sensors easily integrable with current superconducting electronics.

ACKNOWLEDGMENTS

We acknowledge J. S. Moodera and J. W. A. Robinson for fruitful comments. F.G. acknowledges the European Research Council under the European Union's Seventh Framework Program (FP7/2007-2013)/ERC Grant agreement No. 615187-COMANCHE for funding. F.G. and P.S. acknowledge MIUR-FIRB2013 – Project Coca (Grant No. RBFR1379UX) for partial financial support. P.S. has received funding from the European Union FP7/2007-2013 under REA Grant agreement No. 630925 – COHEAT. A.B. thanks the support of the MIUR-FIRB2012 - Project HybridNanoDev (Grant No. RBFR1236VV). The work of F.S.B was supported by the Spanish Ministerio de Economía y Competitividad (MINECO) through the Project No. FIS2014-55987-P and Grupos Consolidados UPV/EHU del Gobierno Vasco (Grant No. IT-756-13).

* francesco.giazotto@sns.it

† sebastian.bergeret@ehu.es

¹ F. Giazotto and F. S. Bergeret, Phase-tunable colossal magnetothermal resistance in ferromagnetic Josephson valves, *Appl. Phys. Lett.* **102**, 132603 (2013).

² F. Giazotto and F. S. Bergeret, Phase-dependent heat transport through magnetic Josephson tunnel junctions, *Phys. Rev. B* **88**, 014515 (2013).

³ P. Machon, M. Eschrig, and W. Belzig, Nonlocal thermoelectric effects and nonlocal Onsager relations in a three-terminal proximity-coupled superconductor-ferromagnet device,

Phys. Rev. Lett. **110**, 047002 (2013).

⁴ A. Ozaeta, P. Virtanen, F. Bergeret, and T. T. Heikkilä, Predicted very large thermoelectric effect in ferromagnet-superconductor junctions in the presence of a spin-splitting magnetic field, *Phys. Rev. Lett.* **112**, 057001 (2014).

⁵ P. Machon, M. Eschrig, and W. Belzig, Giant thermoelectric effects in a proximity-coupled superconductor/ferromagnet device, *New Journal of Physics* **16**, 073002 (2014).

⁶ The thermoelectric effect predicted in Refs.³⁻⁵ has been confirmed in a recent experiment: S. Kolenda, M. J. Wolf, and D. Beckmann, Observation of thermoelectric currents in

- high-field superconductor-ferromagnet tunnel junctions, eprint arXiv:1509.05568 (2015).
- ⁷ F. Giazotto, J. W. A. Robinson, J. S. Moodera, and F. S. Bergeret, Proposal for a phase-coherent thermoelectric transistor, *Appl. Phys. Lett.* **105**, 062602 (2014).
 - ⁸ F. Giazotto, T. T. Heikkilä, and F. S. Bergeret, Very large thermophase in ferromagnetic Josephson junctions, *Phys. Rev. Lett.* **114**, 067001 (2015).
 - ⁹ F. Giazotto, T. T. Heikkilä, A. Luukanen, A. M. Savin, and J. P. Pekola, Opportunities for mesoscopies in thermometry and refrigeration: Physics and applications, *Rev. Mod. Phys.* **78**, 217 (2006).
 - ¹⁰ S. Kawabata, A. Ozaeta, A. S. Vasenko, F. W. J. Hekking and F. S. Bergeret, Efficient electron refrigeration using superconductor/spin-filter devices, *App. Phys. Lett.* **103**, 032602 (2013).
 - ¹¹ J. Linder and J. W. A. Robinson, Superconducting spintronics, *Nat. Phys.* **11**, 307 (2015).
 - ¹² D. R. Schmidt, C. S. Yung, and A. N. Cleland, Nanoscale radio-frequency thermometry, *Appl. Phys. Lett.* **83**, 1002 (2003).
 - ¹³ S. Gasparinetti, M. J. Martínez-Pérez, S. De Franceschi, J. P. Pekola, and F. Giazotto, Nongalvanic thermometry for ultracold two-dimensional electron domains, *Appl. Phys. Lett.* **100**, 253502 (2012).
 - ¹⁴ P. Torresani, M. J. Martínez-Pérez, S. Gasparinetti, J. Renard, G. Biasiol, L. Sorba, F. Giazotto, and S. De Franceschi, Nongalvanic primary thermometry of a two-dimensional electron gas, *Phys. Rev. B* **88**, 245304 (2013).
 - ¹⁵ T. Faivre, D. Golubev, and J. P. Pekola, Josephson junction based thermometer and its application in bolometry, *J. Appl. Phys.* **116**, 094302 (2014).
 - ¹⁶ S. Gasparinetti, K. L. Viisanen, O.-P. Saira, T. Faivre, M. Arzeo, M. Meschke, and J. P. Pekola, Fast Electron Thermometry for Ultrasensitive Calorimetric Detection, *Phys. Rev. Appl.* **3**, 014007 (2015).
 - ¹⁷ F. Giazotto, T. T. Heikkilä, G. Pepe, P. Heliö, A. Luukanen, and J. P. Pekola, Ultrasensitive proximity Josephson sensor with kinetic inductance readout, *Appl. Phys. Lett.* **92**, 162507 (2008).
 - ¹⁸ J. Govenius, R. E. Lake, K. Y. Tan, V. Pietilä, J. K. Julin, I. J. Maasilta, P. Virtanen, and M. Möttönen, Microwave nanobolometer based on proximity Josephson junctions, *Phys. Rev. B* **90**, 064505 (2014).
 - ¹⁹ T. Faivre, D. S. Golubev, and J. P. Pekola, Andreev current for low temperature thermometry, *Appl. Phys. Lett.* **106**, 182602 (2015).
 - ²⁰ M. Silaev, P. Virtanen, T. T. Heikkilä, and F. S. Bergeret, Spin Hanle effect in mesoscopic superconductors, *Phys. Rev. B* **91**, 024506 (2015).
 - ²¹ A. Barone and G. Paterno, *Physics and Applications of the Josephson Effect* (Wiley-Interscience, New York, 1982).
 - ²² M. Tinkham, *Introduction to superconductivity* 2nd ed. (McGraw-Hill, New York, 1996).
 - ²³ R. C. Dynes, J. P. Garno, G. B. Hertel, and T. P. Orlando, Tunneling study of superconductivity near the metal-insulator transition, *Phys. Rev. Lett.* **53**, 2437 (1984).
 - ²⁴ J. S. Moodera, T. S. Santos and T. Nagahama, The phenomena of spin-filter tunnelling, *J. Phys.: Condens. Matter* **19**, 165202 (2007).
 - ²⁵ P. G. de Gennes, *Phys. Letters* **23**, 10 (1966).
 - ²⁶ T. Tokuyasu, J. A. Sauls, and D. Rainer, Proximity effect of a ferromagnetic insulator in contact with a superconductor, *Phys. Rev. B* **38**, 8823 (1988).
 - ²⁷ R. Meservey and P. M. Tedrow, Spin-polarized electron tunneling, *Phys. Rep.* **238**, 173 (1994).
 - ²⁸ X. Hao, J. S. Moodera, and R. Meservey, Spin-filter effect of ferromagnetic europium sulfide tunnel barriers, *Phys. Rev. B* **42**, 8235 (1990).
 - ²⁹ While thermalization of the N layer is, in general, a simple issue due to the finite electron-phonon coupling existing in metals, in a superconducting layer the situation is more subtle due to the exponentially small electron-phonon interaction at low temperature. To this end, *quasiparticle-traps*, i.e., normal metal fingers tunnel-coupled to the S layer⁹ are often used to provide effective thermalization of a superconductor at low temperature by removing hot quasi-particles originating in the nearby-connected overheated N layer.
 - ³⁰ B. Li, G.-X. Miao, and J. S. Moodera, *Phys. Rev. B* **88**, Observation of tunnel magnetoresistance in a superconducting junction with Zeeman-split energy bands, 161105(R) (2013).
 - ³¹ B. Li *et al.*, Superconducting spin switch with infinite magnetoresistance induced by an internal exchange field, *Phys. Rev. Lett.* **110**, 097001 (2013).
 - ³² Y. M. Xiong, S. Stadler, P. W. Adams, and G. Catelani, *Phys. Rev. Lett.* **106**, 247001 (2011).
 - ³³ T. J. Liu, J. C. Prestigiacomo, and P. W. Adams, Spin-Resolved Tunneling Studies of the Exchange Field in EuS/Al Bilayers, *Phys. Rev. Lett.* **111**, 027207 (2013).
 - ³⁴ M. J. Wolf, C. Sürgers, G. Fischer, and D. Beckmann, Spin-polarized quasiparticle transport in exchange-split superconducting aluminum on europium sulfide, *Phys. Rev. B* **90**, 144509 (2014).
 - ³⁵ T. S. Santos *et al.*, Determining exchange splitting in a magnetic semiconductor by spin-filter tunneling, *Phys. Rev. Lett.* **101**, 147201 (2008).
 - ³⁶ Sh. Kogan, *Electronic noise and fluctuations in solids* (Cambridge University Press, 1996).
 - ³⁷ D. Golubev and L. Kuzmin, Nonequilibrium theory of a hot-electron bolometer with normal metal-insulator-superconductor tunnel junction, *J. Appl. Phys.* **89**, 6464 (2001).
 - ³⁸ D. Saint-James, D. Sarma, and E. J. Thomas, *Type II Superconductivity* (Pergamon, New York, 1969); A. I. Buzdin, Proximity effects in superconductor-ferromagnet heterostructures, *Rev. Mod. Phys.* **77**, 935 (2005).
 - ³⁹ J. P. Pekola, T. T. Heikkilä, A. M. Savin, J. T. Flyktman, F. Giazotto, and F. W. J. Hekking, Limitations in cooling electrons using normal-metal-superconductor tunnel junctions, *Phys. Rev. Lett.* **92**, 056804 (2004).
 - ⁴⁰ J. P. Pekola, V. F. Maisi, S. Kafanov, N. Chekurov, A. Kemppinen, Yu. A. Paskin, O.-P. Saira, M. Möttönen, and J. S. Tsai, Environment-assisted tunneling as an origin of the Dynes density of states, *Phys. Rev. Lett.* **105**, 026803 (2010).
 - ⁴¹ O.-P. Saira, A. Kemppinen, V. F. Maisi, and J. P. Pekola, Vanishing quasiparticle density in a hybrid Al/Cu/Al single-electron transistor, *Phys. Rev. B* **85**, 012504 (2012).
 - ⁴² M. J. Wengler, B. Guan, and E. K. Track, Fifth International Symposium on Space Terahertz Technology. **1** 226 (1994).
 - ⁴³ P. Barbara, A. B. Cawthorne, S. V. Shitov, and C. J. Lobb, Stimulated emission and amplification in Josephson junction arrays, *Phys. Rev. Lett.* **82**, 1963 (1999).